Homework, Day 1: Find the first derivative for each function.

SHOW ALL WORK ON ANOTHER PIECE OF PAPER.

1. $f(x)=x^2(2x+5) = 2x^3 + 5x^2$
   $f'(x) = 6x^2 + 10x$
2. $f(x)=(2x-3)^3$
   $f'(x) = 6(2x-3)^2$
3. $y=\dfrac{2x-1}{x^2+1}$ $\quad f'(x) = \dfrac{-2x^2+2x+2}{(x^2+1)^2}$
4. $y=(3x^2-2x+1)(2x-1)$
   $y' = 18x^2 - 14x + 4$
5. $f(x)=(3-4x)^4$
   $f'(x) = 4(3-4x)^3(-4)$
   $= -16(3-4x)^3$
6. $g(x)=(2x^2+3x-2)^2$
   $g'(x) = 2(4x+3)(2x^2+3x-2)$
7. $y=\dfrac{x^2}{x-1}$ $\quad y' = \dfrac{x^2-2x}{(x-1)^2}$
8. $y=(1+x^2)^{\frac{1}{2}}$ $\quad y' = \frac{1}{2}\cdot(1+x^2)^{-1/2}\cdot 2x$
   $= \dfrac{x}{\sqrt{1+x^2}}$
9. $y=\sqrt{2x-1}$
   $y' = \dfrac{1}{\sqrt{2x-1}}$
10. $y=(x-1)^3(x^2+1)$ $\quad y' = (x-1)^2(5x^2-2x+3)$
11. $y=u^3-3u-70$ $\quad \dfrac{dy}{du} = 3u^2 - 3$ $\quad \dfrac{du}{dx} = 15$
    $u=15x+3$ $\quad \dfrac{dy}{dx} = (3u^2-3)(15) = 45u^2 - 45$
    find $\dfrac{dy}{dx}$ $\quad = 45(15x+3)^2 - 45 =$
    $10125x^2 + 4050x + 360$

Analyzing functions using the derivative and the P, Q & PR Rules:

Example #1: Consider the following *rational* function: $f(x)=\dfrac{3x-1}{x^2}$

a. Find the zero(s) of the function. $\quad 3x-1=0$, $x = \frac{1}{3}$

b. Find the vertical asymptote(s). $\quad x = 0$

c. Find the horizontal asymptote(s). $\quad y = 0$

d. Find the relative max and/or min.

$$f'(x) = \frac{3(x^2) - 2x(3x-1)}{x^4}$$

$$= \frac{3x^2 - 6x^2 + 2x}{x^4}$$

$$= \frac{-3x^2+2x}{x^4} = \frac{\cancel{x}(-3x+2)}{\cancel{x^4}x^3} = \frac{-3x+2}{x^3}$$

MAX when $x = \frac{2}{3}$

$f\left(\frac{2}{3}\right) = \frac{9}{4}$ $\quad$ plug into original equation

$\boxed{\left(\frac{2}{3}, \frac{9}{4}\right)}$

e. For what values of x is the function increasing? Decreasing?

$-3x+2$	+		+		−
x^3	−		+		+
	−	0	+	$\boxed{\frac{2}{3}}$	−
	Dec.	(V.A.)	INC	(zero)	Dec

f. Find the point(s) of inflection.

$$f''(x) = \frac{(-3)(x^3) - (3x^2)(-3x+2)}{(x^3)^2}$$

$$= \frac{-3x^3 + 9x^3 - 6x^2}{x^6} = \frac{6x^3 - 6x^2}{x^6} = \frac{6(x-1)}{x^4}$$

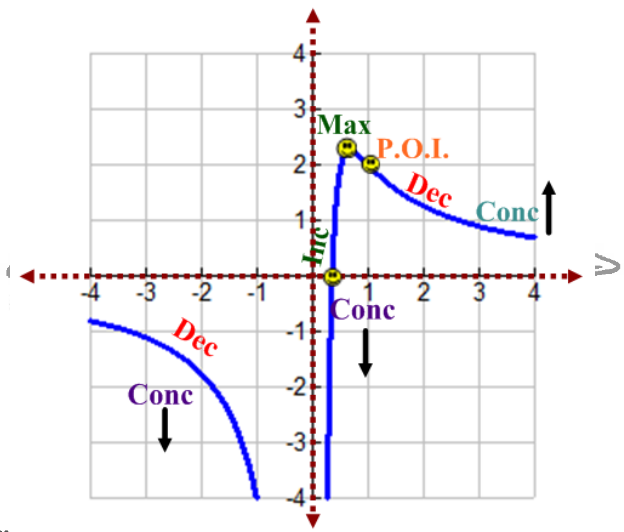
$x = 1$ $f(1) = 2$

$(1, 2)$ P.O.I.

g. Determine concavity.

$6(x-1)$	$-$	$-$	$+$
x^4	$+$	$+$	$+$
	$-$	0 $-$	$+$ Up
	Down	Down	

h. Sketch a graph of the function.



***After you are done sketching your graph, check it with your calculator.

Example #2: Consider the following polynomial function: $f(x) = (4x^2 - 8x + 3)^4$

a. Write the equation of the line tangent to the graph of the function at the point (2, 81).

$f'(x) = 4(4x^2 - 8x + 3)^3(8x - 8)$

$f'(2) = 4(4(2)^2 - 8(2) + 3)^3(8 \cdot 2 - 8)$

$= 864$ slope!

$f(2) = 81$ $\quad 81 = 864(2) + b$

$b = -1647$

$\boxed{y = 864x - 1647}$

b. What are the coordinates of the points where the function has horizontal tangents?

$0 = 4(4x^2 - 8x + 3)^3(8x - 8)$

$= 4[(2x - 3)(2x - 1)]^3 \cdot 8 \cdot (x - 1)$

x = 3/2 $\qquad$ x = 1/2 $\qquad$ x = 1

$f(\frac{3}{2}) = 0$ $\qquad$ $f(\frac{1}{2}) = 0$ $\qquad$ $f(1) = 1$

$(3/2, 0)$ $\qquad$ (1/2, 0) $\qquad$ (1, 1)